Chapter 4 Learning and Teaching

This chapter provides guidelines for effective learning and teaching of the Mathematics Curriculum. It is to be read in conjunction with Booklet 3 in the *Senior Secondary Curriculum Guide* (CDC, 2007), which provides the basis for the suggestions about learning and teaching set out below.

4.1 Knowledge and Learning

Mathematical knowledge is different from knowledge in any other discipline. Its basic assertions have an extremely high degree of certainty, with permanent discipline structures such as axiomatic deductive systems. Unlike theories in empirical sciences, theorems in mathematics are often regarded as paradigms of necessary truths and a priori knowledge. In addition to this epistemological perspective, mathematical knowledge can be perceived as a repository of competencies in knowing certain results and in seeing connections. Viewing mathematical knowledge as products of an individual's conceptual organisation of his/her experience highlights the changing nature of the teacher's role. Teachers become collaborators and facilitators of learning. They should no longer focus on dispensing "truth" or "knowledge", but rather on helping and guiding students to develop their own personalised understanding of mathematics as a result of the activities in which they participate. The learning and teaching of mathematics, therefore, involves "doing" – that is, inquiring, experimenting, abstracting, generalising or conjecturing – not just the transmission and memorising of well constructed knowledge. Teachers are responsible for creating a learning environment in which students develop positive attitudes to learning the subject and are free to express their ideas through active participation.

Another personal or internal conceptualisation of mathematics sees mathematical knowledge as resulting from social interaction; that is, the learning of mathematics involves acquiring facts, concepts, algorithms, principles and skills as a result of social interaction that relies heavily upon context. This approach focuses on students' ability to relate mathematics to its applications and its potential use in problem-solving situations. If students learn through applying mathematics in context, they will come to appreciate the value of mathematics in social settings. It is this "sense making" in the learning of mathematics that enables them to participate actively in "doing

mathematics".

4.2 Guiding Principles

The following are guiding principles and a general rationale for the effective learning and teaching of senior secondary Mathematics.

- **Knowledge**: In Mathematics Education, two kinds of knowledge have been widely discussed, namely procedural knowledge and conceptual knowledge. Students can solve mathematical tasks efficiently through acquiring procedural knowledge; but merely learning procedural knowledge and developing procedural skills do not constitute a balanced mathematics curriculum. It is through acquiring conceptual knowledge that students gain a deeper understanding of mathematics, which then helps them to make connections among different pieces of knowledge. It is important to focus on both kinds of knowledge in the learning of mathematics.

- **Learning**: Students in Hong Kong have a sound basic knowledge and strong procedural skills in mathematics. A number of cross-national studies have shown consistently that Hong Kong students perform very well on tasks routinely learned in schools, but are not so effective in solving exploratory problems and performing hands-on activities. In this regard, the "Inquiry and Investigation" learning units of the Compulsory Part, Module 1 and Module 2 provide opportunities for students to discover and construct knowledge, and to improve their ability to inquire, communicate, reason and conceptualise mathematical concepts through various learning activities. Students are asked to integrate what they have learned in the curriculum to explore and solve more sophisticated real-life problems in the "Further Applications" learning unit in the Compulsory Part. It should be pointed out that the learning *process* and the learning *outcomes (products)* are of equal importance. It is essential to help students move from being passive recipients of knowledge to seeing the relationships between concepts, applying ideas, and ultimately thinking critically and creatively and constructing knowledge on their own.

- **Learning targets**: It is essential to set clear learning targets. Learning will be more effective if students and their teachers have clear learning targets in mind.

- **Teaching for understanding**: A key element in a successful Mathematics lesson is the use of appropriate methods that motivate students to use what they know to act

and think flexibly and to inquire critically to obtain new knowledge. To achieve this, good teaching should involve students' active participation in the learning process, so that they come to enjoy mathematics, recognise its usefulness and appreciate the subject in its own right.

- **Prior knowledge and experience**: It is necessary to pay close attention to the mathematical knowledge, skills and experience that students bring to the classroom. Teachers should therefore have a good understanding of their students' strengths, interests and needs.

- **A wide range of pedagogies**: There are many strategies for teaching Mathematics, some of which supplement each other. Teachers should consider the flexible use of a variety of teaching strategies.

- **Quality interaction**: The effective use of questioning and feedback guides students in the learning process. Students also contribute to each other's learning when they interact in different groupings to give mutual feedback. Teachers may use "scaffolds" to support them in areas that may be beyond their current reach.

- **Teaching for independent learning**: Generic skills (such as critical-thinking skills and problem-solving skills) and reflection should be nurtured through relevant learning activities. Students at the senior secondary level are more mathematically mature and should be encouraged to read mathematics-related articles or books and gather information on their own to widen their knowledge base.

- **Self-directed learning**: Students should be encouraged to take responsibility for their own learning. Open-ended questions can be used to promote their interest in learning mathematics. Opportunities should be provided to enable students to plan their own learning and to use mathematics to solve real-life problems.

- **Feedback and assessment**: Teachers need to assess the work of students, but assessment has little value without feedback. Feedback should not be confined to giving a grade, but must also provide students with a wide range of information on strengths and areas where they can improve their learning. Evidence from assessment should also give teachers indications as to where they need to improve their teaching.

- **Resources**: In planning and conducting mathematical activities, teachers should make use of a variety of learning and teaching resources, such as textbooks, reference books, audio-visual materials and instruments for drawing shapes and making models. Well-designed learning materials play a crucial role in providing students with a means of working at their own pace, so that they can reach and, in some cases, move beyond the skills and understandings already achieved by the majority of their peers. Commonly-used IT tools and hand-held technologies, such as computer-aided learning packages, dynamic geometry packages and graphing calculators, can be used to promote learning and teaching.

- **Motivation**: In general, Hong Kong students have a high regard for Mathematics as a school subject. However, a low level of confidence, a lack of perseverance in tackling difficult problems and a reluctance to discuss their learning difficulties with teachers are common barriers for them when learning mathematics. Effective motivational strategies need to be adopted to overcome these barriers. For instance, motivation can be aroused by giving students positive reinforcement and by providing them with opportunities to express their views on learning the subject. It can also be aroused by enabling them to apply their mathematical knowledge and skills successfully in real-life situations.

- **Engagement**: Students should be encouraged to participate actively in activities, collaborate closely with others, express themselves openly and treat suggestions positively, and through all of these to build up their self-confidence and positive attitudes towards learning – despite the risk of making mistakes or encountering difficulties.

- **Catering for learner diversity**: Students have a wide variety of characteristics and strengths. Teachers should employ a range of strategies to cater for such learner diversity.

4.3 Choosing Learning and Teaching Approaches and Strategies

The extent to which teachers are successful in facilitating learning among students depends, to some degree, on the teaching approaches and strategies they use. A variety of learning and teaching approaches can be interwoven and deployed to suit students with different needs, interests, abilities and prior knowledge. Familiarity with the characteristics of Hong Kong students in Mathematics can help teachers adopt the most

suitable learning and teaching strategies. In general, Hong Kong students:

- have a high regard for Mathematics;
- believe that diligence can help them to learn mathematics;
- are strong in basic skills and computation, but are weaker in dealing with non-routine problems;
- value drill and practice;
- are more used to exposition than activity-based ways of learning such as the use of games or group discussion;
- seldom interrupt the flow of teaching by asking questions; and
- are usually motivated by extrinsic factors.

Three common pedagogical approaches that can be employed for the effective delivery of the Mathematics Curriculum (S4 – 6) are discussed below.

Teaching as direct instruction

Direct instruction, a very frequently used approach in the Mathematics classroom, can make a positive contribution to mathematics learning if it is dynamic, and well planned and organised.

A direct instruction approach is most relevant to contexts which involve explanation, demonstration or modelling to enable learners to gain knowledge and understanding of particular concepts. In this approach, the teacher focuses on directing the learning of the students through providing information or explanations. Direct instruction, in which teachers present factual materials in a logical manner, can be employed effectively for teaching definitions and notations of mathematical terms, rigorous proofs of mathematics theorems and procedures for sketching curves. This approach can also be used with a large group of students to stimulate thinking at the start of an open discussion. Teaching mathematics through direct instruction can produce a mathematically neat and complete lesson which contains presentations and explanation leading to an intended conclusion. Although it can be interactive, it is a more teacher-centred approach: the teacher asks questions, chooses students to answer them and evaluates the responses; and then he/she probes for more information and may ask the students to justify their answers.

Another major goal of direct instruction is to maximise student learning time. In a typical direct instruction lesson on Mathematics, the teacher usually spends some time lecturing, guides the students through a concept, provides examples and breaks complex ideas into simpler steps. The students are then given simple problems to try on

their own; and the simple problems are then combined so that students can solve more complex problems. Finally, the teacher summarises what the students should have learned, and gives them assignments through which he/she can assess the amount of the content the students have learned in that lesson. Direct instruction lessons can also be facilitated by the use of audio-visual materials, when appropriate.

Teaching the concept and notation of Combination using the direct instruction approach

Students are introduced to the concept of Combination by their teacher. The teacher explains the concept and notation of Combination and puts the related formulae on the board. The teacher then derives the formulae and demonstrates with some examples. The difference between the concepts of Combination and Permutation is also clearly explained. The teacher then alerts the students to the fact that the terminology of Combination in mathematics is different from its common use in daily life – for example, the code of a "combination lock" does not refer to Combination but to Permutation. After that, the teacher introduces other notations for Combination used in various parts of the world. At the end of the lesson, a brief note on what has been taught and some simple exercises on the topic are distributed to the students for consolidation.

The "direct instruction" approach is suitable for teaching the concept and notation of Combination for the following reasons:

- The concept and the notation are difficult for students to understand and the proofs of the formulae are mainly procedural.
- It can be ensured that students have acquired an accurate understanding of the concept and notation.
- The notations used in the books from different countries may be different. For instance, the notations of Combination $\binom{n}{r}$ and C_n^r are usually found in the books written by British and Chinese scholars respectively. Students will be confused by the notations if this is not explained properly.
- The concept and notation of Combination can be introduced efficiently in a short time.

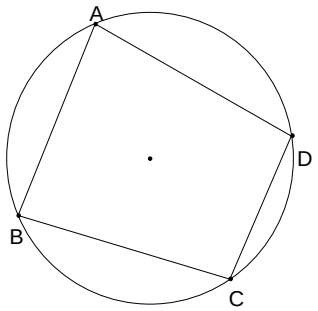
Teaching as inquiry

The Mathematics Curriculum (S4 – 6) sets out various learning activities for students to carry out investigations and includes "Inquiry and Investigation" learning units in the Compulsory Part, Module 1 and Module 2. In the inquiry approach, the emphasis is on

the process and the action undertaken by the learner – inquiry tasks often focus on students' thinking and inquiring processing. Students are encouraged to help each other raise questions and develop understanding, with their experiences being seen more as a resource for learning than as a product. Students engage in complex cognitive processes requiring thoughtful discourse. They are invited, for example, to **make connections between facts, make predictions, and debate alternatives**. This relies heavily upon **extensive dialogue** among students, and can take place during **interactive whole-class teaching or during peer interaction in pairs or groups**. This approach promotes critical thinking and problem-solving skills. The topic(s) under investigation should be placed in a meaningful context, and students should be encouraged to explore and discover information through the use of **open-ended questions, group discussion, investigations, experiments and hands-on exercises**. However, there should be sufficient "wait time", so that students can explain their thinking adequately and elaborate their answers.

Teaching properties of a cyclic quadrilateral using the inquiry approach

The teacher starts the topic "properties of a cyclic quadrilateral" in the Compulsory Part by asking students to use computers with dynamic geometry software installed and an application file with a circle and a cyclic quadrilateral drawn as in the figure below.



Students are free to:

- move the vertices of A, B, C and D;
- add lines such as diagonals AC and BD;
- extend line segments AB, CD, etc.;
- measure sizes of angles; or
- measure lengths of line segments such as AB, AC, etc.

Students are asked to record their measurements and make conjectures about the properties such as angle relations or line relations. They are encouraged to discuss their

observations and conjectures with each other, and may use the software to provide evidence for them. After that, students are asked to make a presentation on their observations and conjectures, during which the teacher focuses on asking students to explain and justify their conjectures and gives them time to do so. For example, if students observe that the opposite angles of a cyclic quadrilateral are supplementary, the teacher may ask them whether they have verified their conjecture by referring to different quadrilaterals. It should be pointed out that students' observations and conjectures may differ and the teacher should provide different responses to students who have arrived at different conclusions.

As students have often encountered circles and quadrilaterals in daily life and in their previous mathematics learning experiences, they can integrate their prior knowledge and IT skills to explore new aspects of plane geometry. This topic allows students to explore a rich collection of relations, including the basic properties listed in the Compulsory Part and also aspects such as Ptolemy's Theorem* that is beyond the curriculum. Through engaging in this exploratory activity, they may come to recognise the richness of geometrical knowledge. They can construct knowledge through observing, making conjectures and testifying. Sketching a diagram using IT tools or drawing instruments are good ways for students to justify their hypotheses and conjectures. This approach is different from the familiar deductive approach to study geometric knowledge.

* Ptolemy's Theorem: For a cyclic quadrilateral, the sum of the products of the two pairs of opposite sides equals the product of the diagonals.

Teaching as co-construction

This approach puts the focus on the class as a community of learners. It is based on the view that mathematics is a creative human activity and that social interaction in the classroom is crucial to the development of mathematical knowledge. The teacher plays a central role in developing a problem-solving environment in which students feel free to talk about mathematics. Tasks for this approach usually involve creating knowledge and developing criteria for judging that knowledge collectively. A wide variety of linkages are made between members of the class as they contribute to the construction of knowledge. In this approach, the teacher helps students learn how to "think for themselves", which may require temporary frameworks or scaffolds such as model

answers, demonstrations, prompts and feedback, opportunities for students to explain their arguments to the class, or worksheets for students to work on. The teacher also requires students to check the process through which they reached a conclusion and indicate how it might be improved.

This approach relies heavily on discussion and sharing among participants. Students learn through student-student and teacher-student interaction. Through co-construction of knowledge, students develop social skills, organise their thinking and develop rational arguments.

Teaching the quadratic formula
using the co-construction approach

In teaching "solving quadratic equations by the quadratic formula", a Mathematics teacher may employ a co-construction approach in which, instead of passively accepting knowledge, students are able to construct concepts and to understand how the formula is deduced. After learning how to solve quadratic equations by the factor method, students are asked to solve quadratic equations with irrational roots; and it is anticipated that they will discover that not all quadratic expressions can be factorised into linear expressions with rational coefficients – and so the factor method is limited in its usefulness. By being actively involved in well-constructed tasks – for instance, from solving $x^2 - 5 = 0$, $x^2 - 2x - 5 = 0$ to $x^2 - 2x - c = 0$ – and class discussion, students should become responsible for explaining their reasoning orally to the class, developing their own analogical thinking and discovering how the method of completing the square can be extended to solving general quadratic equations. The teacher has to sum up the discussion and refine the constraints (e.g. the coefficient of the term involving x^2 must be non-zero) in collaboration with students.

- In the co-construction approach, the teacher focuses on making connections between facts and fostering new understanding in students. The teacher encourages students to analyse, interpret and predict information. He/She also promotes extensive dialogue among students, which is a very important part of learning mathematics. Communication is a way of sharing ideas and clarifying understanding. When students think, discuss and inquire about mathematical concepts, they reap a dual benefit: they communicate to learn mathematics, and they learn to communicate mathematically.

- Teaching is developmental rather than merely directive or presentational.

Although algorithms provide an efficient route to getting correct answers, students do not understand how or why these procedures work. Merely directing them to follow the traditional rules or algorithms in a fixed fashion is like expecting them to arrive without having made the journey. The co-construction approach is student-oriented: the teacher serves as an expert providing guidance and does not dictate the working strategies entirely. Lessons should begin from where the students are, not from where the teacher is. Students are expected to bring the necessary skills and background knowledge to the task and to be self-motivated. Individual differences are expected, welcomed and supported.

- The classroom climate is collegial, supportive and spontaneous. Discussion, group work and active participation are encouraged and expected. Students are active creators of knowledge, not passive receivers of information. The emphasis is on understanding rather than on memorisation and repetition. Activities are problem-centred and student-driven. Lessons are built on solving problems at the appropriate level for the students. Some easier problems are dealt with early in the lesson and are used to provide paradigms. Busy work and unnecessary repetition are minimised.

As mentioned earlier, in the learning and teaching of mathematics, a single strategy is seldom adopted: an effective teacher of Mathematics very often integrates various strategies when teaching a topic. Strategies should always be used flexibly and be suited to the abilities, interests and needs of students, and the context and development of the lesson. Some common classroom practices which are effective for learning and teaching mathematics include:

- expanding concepts or definitions;
- analysing a problem in more than one way;
- using an idea in more than one setting;
- providing models that enhance understanding;
- giving examples of applications;
- providing examples and counter-examples when explaining concepts;
- requesting students to rephrase problems;
- practising and drilling;
- using scaffolding to demonstrate how to tackle a problem by, for example, induction, deduction, analogy, looking for equivalence and identities, classification, conjecturing, generalising, exhausting, formulating hypotheses,

looking for patterns and regularities, charting and graphing, abstracting, intuition, analysing and synthesising, and breaking down a complicated problem into smaller parts;

- posing problems by students through changing the knowns, unknowns or constraints;
- providing opportunities for discussion and sharing of ideas; and
- brainstorming among students.

The example below illustrates how some of the approaches and strategies can be used in the Mathematics classroom:

Teaching one of the properties of the scalar product of vectors using the direct instruction, the inquiry and the co-construction approaches

Teachers may integrate various teaching approaches and classroom practices to introduce the properties of the scalar product of vectors so that the lessons can be more vivid and pleasurable. In this example, teaching one of the properties of the scalar product of vectors, $|\mathbf{a} - \mathbf{b}|^2 = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2(\mathbf{a}\cdot\mathbf{b})$, is used as an illustration.

In previous lessons, the teacher has taught the concepts of magnitudes of vectors and the scalar product of vectors using **direct instruction**. In this lesson, the students are divided into small groups to promote discussion, and the groups are asked to **explore** the geometrical meaning of the property. Here, the **inquiry approach** is adopted, with students having to carry out **investigations** with the newly acquired knowledge related to vectors. During the **exploration**, the groups may interpret the geometrical meaning differently. Some may consider one of the vectors to be a zero vector and get the above property; but others may relate it to the Pythagoras' Theorem by constructing two perpendicular vectors **a** and **b** with the same initial point. Hence, the hypotenuse is $|\mathbf{a} - \mathbf{b}|$ and $\mathbf{a}\cdot\mathbf{b} = 0$ and the result is then immediate. If some groups arrive at this conclusion, the teacher should **guide them to discover** that their interpretation is only valid for special cases. However, the geometrical meaning of this property is related to the cosine formula learned in the Compulsory Part. If some groups can find that the property is the vector version of the cosine formula, they can be **invited to explain** how they arrived at this geometrical meaning. If none of the groups can arrive at the actual meaning, the teacher may **guide** them to find it out by **giving prompts**. Some **well-constructed prompts (or scaffolds)**, such as asking them to draw various types of triangles and **find clues** to connect $|\mathbf{a}-\mathbf{b}|$, $\mathbf{a}\cdot\mathbf{b}$, $|\mathbf{a}|$ and $|\mathbf{b}|$ with the triangles drawn, may be provided. The **co-construction approach** is adopted here.

After understanding the geometrical meaning, the result can be derived by applying the cosine formula learned in the Compulsory Part. The groups are further asked to **explore** alternative proofs. Here, the **inquiry approach** is employed. The groups may not think of proving this property with $|\mathbf{x}|^2 = \mathbf{x}\cdot\mathbf{x}$ directly. The teacher may **give some hints to guide** them. In this case, the teacher and the students are **co-constructing** knowledge. If the students still cannot prove this property, the teacher can **demonstrate** the proof on the board using the **direct instruction approach**. Whatever methods the students use, they are **invited to explain** their proofs to the class. During the explanation, the teacher and students may **raise questions and query** the reasoning.

4.4 Classroom Interaction

Classroom interaction helps students construct knowledge with the support of teachers and peers. Students need to participate in the discussion of concepts, procedures and proofs in the Mathematics classroom. They also have to think, listen and speak as they participate in a particular mode of inquiry. It is not enough for students simply to listen to the teacher and answer teachers' questions. Students need to develop competencies for critical thinking, and in order to develop these they need to have opportunities to think and interact with others.

Classroom interaction can be fostered in various ways, such as the following:

- *Begin each class with a problem*
  Instead of simply saying "We are going to cover the quadratic formula in this lesson", a teacher can begin by asking "Do we have a general formula for solving quadratic equations?"

- *Be patient and use silence to encourage reflection*
  There should be pauses after questions to encourage student responses. The teacher should wait for students' replies, and resist the temptation to break the silence by answering the questions for them.

- *Arrange and use classroom space to encourage interaction*
  Putting students into small groups or arranging them to sit face to face in a circle

may promote discussion and interaction among them.

- *Create a friendly environment*
  The teacher should welcome students' ideas and consider all their views without rushing into making judgments. He/she should also focus discussion on ideas rather than on judging people.

How teachers view and handle interaction is a key factor in determining the effectiveness of classroom interaction. Making use of effective questioning techniques, providing an encouraging classroom environment, scaffolding and providing appropriate feedback are important for maintaining the sort of interaction that leads to learning.

(a) *Questioning*

Appropriate questioning practices not only elicit students' thinking but also enhance their understanding. The questions the teacher asks can take various forms. Sometimes they may be simple, lower-order questions, for which students may have an immediate answer, to check whether they have learned factual information. At other times the teacher has to ask more open questions to which there may not be any one or simple answer. For example, it is helpful for the teacher to ask: "Can you explain how you got this answer?" or "Could you make use of a diagram to help with your explanation?" Instead of merely giving correct answers, the teacher has to be a good listener and give students sufficient time and opportunities to build up their understanding of mathematics through contextualised problems that make sense to them. This allows the teacher to gather useful information on students' progress in understanding mathematical ideas.

Tips for questioning in the classroom

- Pause (giving students time to think) after asking a question.
- Avoid asking questions which require "yes" or "no" answers.
- Follow up student responses by asking "why" or by passing the question to the class or another student for a reaction.
- Limit the use of questions that rely almost completely on memory.
- Avoid directing a question to students for disciplinary reasons.
- Avoid asking questions that contain the answer.
- Do not call on a particular student before asking a question.

- Ask open-ended questions.
- Do not label the degree of difficulty of a question.
- Avoid asking for choral responses from the whole group.

(b) *Scaffolding*

One of the teacher's roles has been conceptualised as "scaffolding". Scaffolding is the support that a teacher provides in helping students learn how to perform a task that they cannot yet manage on their own. In Mathematics, teachers' scaffolding practices can be categorised into three levels. Level 1 scaffolds refer to those prompts and stimuli that already exist in the environment, either as a result of deliberate planning or by chance, and which serve to support students' learning in Mathematics. These may include, for instance, a poster displaying useful formulae, and computer manipulatives such as dynamic geometry software and self-correcting mathematical games. Although the teacher's immediate involvement may be low, the level of support can be high depending on the time and effort he/she has expended in finding out which displays, tasks and materials are available. Level 2 scaffolds include direct interactions between teachers and students focusing on the specific tasks in hand. Practices that require students to be involved more actively in the learning process are also included in scaffolding at this level. The strategies adopted may vary from direct instruction, showing and telling, to more collaborative meaning-making. Level 3 scaffolds aim to make connections between students' prior knowledge and experience and the new mathematics topics to be learned. Students are likely to engage in longer, more meaningful discussions, and meanings are more easily shared when each student engages in the communal act of making mathematical meaning. The two Further Learning Units – "Further Applications" and "Inquiry and Investigation" – aim at providing teachers with a platform for providing scaffolding at this level.

It is worth noting that the approach of encouraging student discussion is heavily dependent on establishing mutual trust between the teacher and the students. On the one hand, the teacher has to trust the students to persist in attempting to solve mathematical problems so that they can feel free to talk about their solutions and describe how they reached them. On the other hand, the students must trust the teacher to respect their efforts and give them opportunities to discuss their understanding and explain their attempts to solve the problems.

Criteria for choosing a mathematical problem

A good mathematical problem:

- has important, useful mathematics embedded in it;
- can be approached in several ways, using different solution strategies;
- has various solutions or allows different decisions or positions to be taken and defended;
- encourages student engagement and discourse;
- requires higher-order thinking and problem-solving skills;
- contributes to students' conceptual development;
- has connections with other important mathematical ideas;
- promotes the skilful use of mathematics;
- provides an opportunity to practise important skills; and
- creates an opportunity for the teacher to assess what his/her students are learning and where they are experiencing difficulties.

4.5 Learning Community

As noted earlier, social interaction in the classroom plays an increasingly important role for students learning mathematics. Interaction between the teacher and students, and among the students, influences what is learned and how it is learned. It is important for the teacher to cultivate a "classroom culture" in which helping one's peers to learn is not a marginal activity, but a central concern. Once such a culture has been established, opportunities for learning arise (which are not usually present in traditional classrooms) as students collaborate to solve problems. Thus, building a "learning community" provides a new way of seeing and managing the classroom to improve the quality of learning and teaching. Presentations to the class after project work or reading tasks can be exciting and challenging, and giving presentations enables students to connect knowledge, skills, values and attitudes when constructing their mathematical knowledge.

In this type of classroom setting, students learn a lot more than Mathematics content. They develop beliefs about mathematics and about their own and the teacher's role. A sense of what is valued is also developed, along with attitudes and forms of motivation. This approach is designed to foster the belief that persisting in figuring out personally challenging problems is valued more than a page of correct answers, and that

cooperation and negotiation have greater value than competition and conflict. Students are encouraged to talk about their methods of solution without their correctness being evaluated. As a result, students share a sense of participation in a learning community, in which, collectively, they are more willing and able to engage in activities and accept the responsibility to co-construct and judge knowledge and understanding within the class.

4.6 Catering for Learner Diversity

Attention should not be focused only on academically lower achievers or those who are more capable in mathematics. The needs of all students, whatever their ability level, are equally important.

Measures to cater for learner diversity at the curriculum level and the school level have been described in Chapters 2 and 3 respectively. In daily classroom teaching, teachers can cater for learner differences by providing students with different assignments or activities graded according to levels of difficulty, so that students work on exercises that match their progress in learning. For less able students, assignments may be relatively basic and straightforward in nature; and for more able students, assignments can be more open-ended and challenging enough to cultivate and sustain their interest in learning. Alternatively, teachers can provide students with the same exercises, but with variations in the amount and style of support given – for example, by providing more clues, or by breaking a complicated problem into several simpler but connected parts for students with weaker performance.

In interacting with the whole class, teachers can make adjustments to suit the needs of students. For example, they can ask less able students simple questions (e.g. If this polynomial is divided by x - 3, what value of x should be put into the polynomial to obtain the remainder?) and reserve more challenging questions for the more able ones (e.g. If you are not allowed to use long division, how can you find the remainder when this polynomial is divided by $x^2 - 3x + 2$?).

Besides whole-class teaching, teachers can consider different grouping strategies to cater for the needs of students and enhance peer-to-peer interaction. Students can also be encouraged to express themselves openly and share their work in groups to build up their self-confidence and knowledge.

Information technology provides a viable means for teachers to cater for learner differences. For some topics, such as graphs of trigonometric functions, curve sketching and definite integrals, the use of appropriate software packages can promote vivid and rigorous learning and teaching, as well as enhancing students' interest in the subject. IT tools can enhance learning for able students by letting them extend their horizons in mathematics beyond their current practice; and they can allow less able students to focus more on strategies for tackling problems, or on drawing conclusions from a large set of data from daily-life contexts, without going through tedious computations and mathematics procedures.

4.7 Use of Information Technology (IT) in Learning and Teaching

Technology has had a fundamental impact on the learning and teaching of mathematics. The memorisation of static facts and the drilling of skills in mathematics are not as important as in the past. Mathematics is now generally considered to be primarily a search for patterns and relationships. When students are exploring regularities, looking for recurring themes and making conjectures, they are doing mathematics.

Commonly used IT tools in the learning and teaching of mathematics include computers, hand-held technologies (such as graphing calculators) and mathematics-oriented software packages. These tools have the potential to support students in their learning and help them construct knowledge in such a way that they gradually become independent and confident in demonstrating the required skills and understanding. Powerful mathematics-related software packages can also help to engage students who at first do not like Mathematics by letting them experience the excitement and enjoyment of learning the subject.
Students in Hong Kong have generally been equipped with basic IT skills in their junior secondary schooling. Teachers can make good use of their proficiency in technology to enhance the learning and teaching of Mathematics by:

- extending mathematics learning experience, and encouraging active student participation in exploratory and investigatory activities;
- supporting, supplementing and extending learning and teaching activities such as exercises and tutorials, charting, graphical analysis, simulation, modelling, information retrieval and data processing;

- developing new teaching strategies and practices in the classroom, such as providing students with an interactive learning environment for contextual and situational learning; and
- contributing to students' self-regulated learning with the help of computer-assisted learning packages and the resources available on the Internet, and extending students' learning outside the classroom.

It is nevertheless necessary to make appropriate decisions on when to use technology (e.g. to know when to use a graphing software or to work by hand to sketch the graph of a rational function at different learning and teaching stages). Teachers have to decide when to use computers or hand-held devices to demonstrate a mathematical concept (e.g. to explore the features of the graphs of quadratic functions using an interactive approach).

In some cases, IT tools may provide shortcuts which are undesirable, especially in a learning process in which mental development is required for concept-building. Exaggerated use of such tools may lead to a de-emphasis on skills. It should be stressed, therefore, that Mathematics should be taught in its own right, with its own educational objectives, for the information age.